

## Lecture 7: Forces and the motion they produce

As we have seen in the last 6 lectures, we use displacement, velocity and acceleration to describe the motion of an object. The study of this motion is called kinematics. Now we begin a study of the forces and the motion that they produce. The methods we have used in studies of kinematics are essential to our work, but now we add another feature, the idea that forces can produce motion. To get started we need to talk about forces.

### **Forces: What are they?**

Forces are traditionally subdivided into “field” forces and “contact” forces. Examples of field forces are gravity and the coulomb force between charged particles. Examples of contact forces are friction or a hammer hitting a nail. However all forces are really field forces if you look closely enough. In fact there are four fundamental field forces: the gravitational force; the electromagnetic force; the weak force; the strong force. The strong force is what holds the nucleus together and the weak force also acts at the nuclear length scale (about a fermi). The forces that dominate our everyday life are the gravitational and electromagnetic force and the forces that are called contact forces are really due to the quantum mechanics of the electromagnetic force of electrons in atoms. To understand dynamics it is very useful to develop the methods based on the idea of contact forces so we don’t have to worry about the details of the motion of electrons near atoms. Since force is so important we give it a separate unit, the Newton (N). This is a derived unit, as it is related to the base units of mechanics: m, kg, s. We will see the relation below.

### **The effect of forces : Newton’s laws**

Note that there is no way to derive these laws. They are laws which were postulated in order to explain experimental observations. They have stood the test of time and are now considered fundamental to all physics. They were written down most completely in Newton’s “Principia” (actually it’s full title is: Philosophiae Naturalis Principia Mathematica) published in 1687 in latin. Principia remains the most profound scientific treatise ever published, with Darwin’s “Origin of Species” published in 1859 an honorable second.

#### *First law*

A body in motion with velocity  $\vec{v}$  remains in motion with velocity  $\vec{v}$ , unless acted upon by an net external force.

#### *Second Law*

A net external force,  $\sum \vec{F}$ , acting on an inertial mass  $m$  produces an acceleration  $\vec{a} = \sum \vec{F}/m$ . This is more commonly written as,

$$\sum \vec{F} = m\vec{a} \tag{1}$$

From this equation we see that the unit of force,  $N = kgm/s^2$ . Note that  $1N = 0.225lb$ , where the  $lb$  is the unit of force in US units. The unit of mass in US units is the slug.

#### *Third Law*

To every action there is an equal and opposite reaction.

### **Applications**

The number of applications of the above simple looking laws is truly staggering and is testament to the fact that there are simple unifying principles underlying much of the behavior we observe in world around us. However though the above laws look simple, solving problems using them can be complex and requires a good physical understanding of what Newton's laws tell us. In fact there are many applications of even the simple case where there is no motion, ie  $\vec{a} = 0$ . In that case Newton's second law tells us that  $\sum \vec{F} = 0$ , so there can be no net force on the object. This case is extremely important in civil engineering, where we want bridges and buildings to stay where they are. The study of cases where the forces sum to zero is called "statics" or "engineering statics". This case also has many biomechanics applications, for example in hip replacements and artificial knees. Dynamics is the study of the case where  $\vec{a} \neq 0$  and is extremely important in mechanical engineering where the design of machines to move and do work is a key task. As we shall note later Newton's laws also underlie fluid dynamics such as blood flow in the body and atmospheric dynamics such as weather patterns and hurricanes.

First lets look at an application to one-dimensional motion:

*Dynamics example.* Consider a stationary mass  $m = 10kg$  on a frictionless surface, located at the origin. At  $t = 0$ , we apply a constant force  $\vec{F} =$

$(F_x, F_y, F_z) = (-6.0N, 0, 0)$  to the mass. Find the position of the mass 5.0s after the force is applied.

*Solution.* From Newton's second law, we have

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{(-6N, 0, 0)}{10kg} \quad (2)$$

Since the acceleration is only in the x-direction, the mass moves only in the x-direction, so that  $a_x = -6/10m/s^2$ . Since the force is constant, so is the acceleration, so we can use our constant acceleration formula.

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2 = (0.50) * (-0.60) * 5.0^2 m = -0.75m \quad (3)$$

The mass is located at  $x = -0.75m$  at time  $t = 5s$ .

Now lets look at an application in statics:

*Statics example.* Consider a gymnast who holds herself in the air, while keeping her arms vertical, by using her two hands to grasp two different rings. The rings are at the same height and are attached to a supporting beam by two different ropes. The first rope makes an angle of  $20^\circ$  to the vertical and the other rope makes an angle of  $-40^\circ$  to the vertical, and the two ropes lie in the same plane. Find the tension in the two ropes, if the gymnast has a mass of  $40kg$ .

*Solution* Since there is no motion, the sum of the forces must be zero. In the x-direction this implies that,

$$T_1 \sin(20^\circ) = T_2 \sin(40^\circ). \quad (4)$$

The sum of the forces in the y-direction gives,

$$T_1 \cos(20^\circ) + T_2 \cos(40^\circ) = 40kg * 9.8m/s^2. \quad (5)$$

From the first equation, we have  $T_2 = T_1 \sin(20^\circ) / \sin(40^\circ) = 0.53T_1$ . Putting this in the second equation we get,

$$1.35T_1 = 392N, \quad (6)$$

so that,  $T_1 = 291N$ . Using this in either of the above equations yields  $T_2 = 155N$ .